Name: ______________________________

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have 75 minutes to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	8	6	9	5	9	9	16	3	0	65
Score:										

1. Suppose f is a function with continuous second partial derivatives. You are given the following information about f and its derivatives at five points:

	$(0,0)$	$(1,1)$	$(2,3)$	$(1,5)$	$(-1,1)$
f	3	0	-1	2	5
f_x	0	0	0	0	0
f_y	0	-1	0	0	0
f_{xx}	1	2	-2	2	-1
f_{xy}	3	-1	1	-1	3
f_{yy}	8	-1	-2	2	2

(a) (5 points) Classify each of the given points as a local maximum, local minimum, saddle point, or none of those.

$(0,0)$ ______________________________

$(1,1)$ ______________________________

$(2,3)$ ______________________________

$(1,5)$ ______________________________

$(-1,1)$ ______________________________

(b) (3 points) Estimate $f(0.9,1.1)$ using the linearization of f (at the most appropriate base point).

2. Find the following limits if they exist. Fully justify your answers.

(a) (3 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4+y^4}{x^2+y^2}$

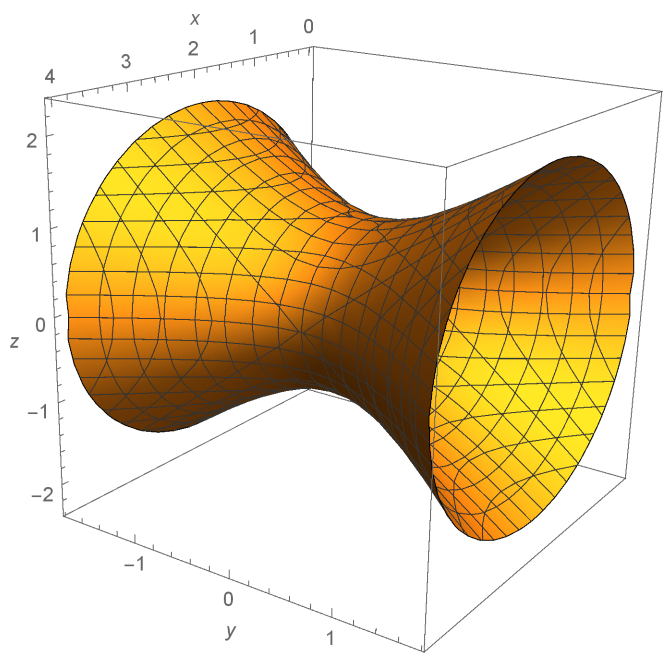
(b) (3 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3-yx^3}{x^4+y^4}$

3. The lines $\ell_1(t) = (3,1,4) + t(-1,-1,1)$ and $\ell_2(t) = (7,0,8) + t(6,1,2)$ **do intersect**.

   (a) (3 points) Find the point where they intersect.

   (b) (6 points) Find an equation of the plane containing both lines.

4. (5 points) Match the given surface to its equation. Also give the name of the surface.



A. $x^2 + y^2 - z^2 = 1$

B. $x^2 + y^2 - z^2 = -1$

C. $x^2 - y^2 + z^2 = -1$

D. $x^2 - y^2 + z^2 = 1$

E. $(x-2)^2 - y^2 + z^2 = -1$

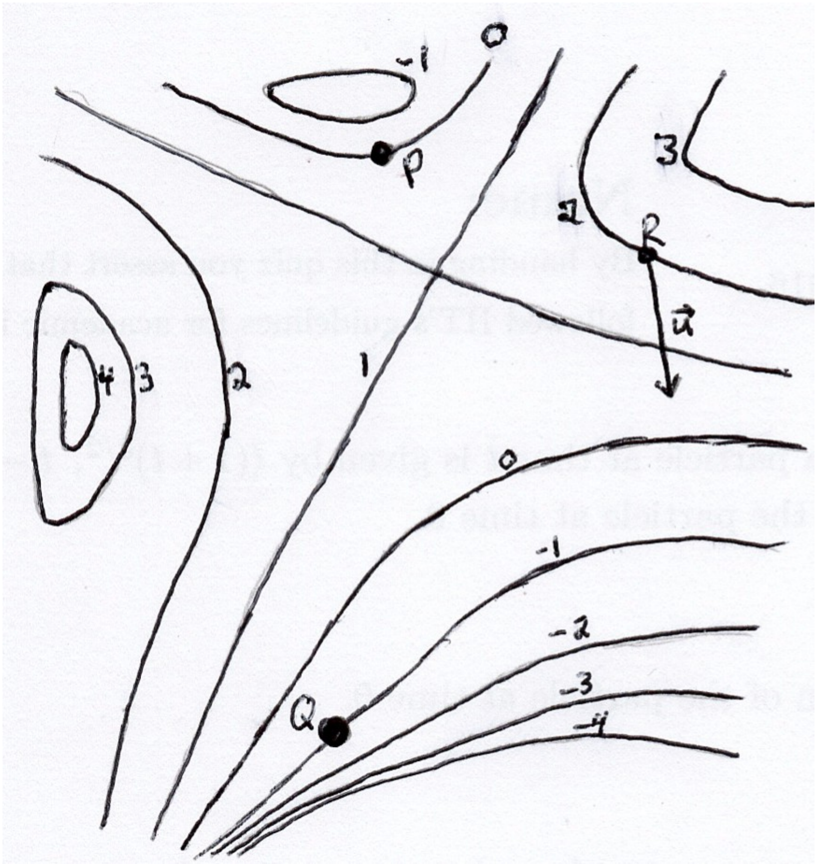
F. $(x-2)^2 + y^2 - z^2 = 1$

G. $(x-2)^2 - y^2 + z^2 = 1$

H. $(x-2)^2 + y^2 - z^2 = -1$

Name of surface: ______________________

5. Below is a contour plot of a function f, together with several points and a unit vector $\mathbf{u}$.



(a) (3 points) Sketch in ∇f at each labelled point.

(b) (2 points) Estimate $D_{\mathbf{u}} f(R)$.

(c) (4 points) Circle below the sign of each second partial derivative of f at Q.

$f_{xx}(Q):$ $+$ $-$ 0

$f_{xy}(Q):$ $+$ $-$ 0

$f_{yy}(Q):$ $+$ $-$ 0

6. Consider the function $f(x, y) = x^2 + 4y^2$, and the disk D given by $x^2 + y^2 \leq 1$.
   (a) (2 points) Find f_x and f_y.

   (b) (5 points) Find the maximum and minimum values of f on D.

   (c) (2 points) Draw D together with any critical points you found in (b) and the level curves of f corresponding to the values at those critical points.

7. (16 points) Circle 'True' or 'False' (2 points each, no partial credit):

(a) True False For any two vectors $\mathbf{u}$ and $\mathbf{v}$, we have $|\mathbf{u} \times \mathbf{v}| = |\mathbf{v} \times \mathbf{u}|$.

(b) True False For any three vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$, we have $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$.

(c) True False For any vector $\mathbf{v}$, we have $|\mathbf{v} \cdot \mathbf{v}| = |\mathbf{v}|^2$.

(d) True False For any vector $\mathbf{v}$, we have $|\mathbf{v} \times \mathbf{v}| = |\mathbf{v}|^2$.

(e) True False For any vector function $\mathbf{r}(t)$, $\left| \int_a^b \mathbf{r}'(t)\, dt \right| = \int_a^b |\mathbf{r}'(t)|\, dt$.

(f) True False If a curve in the plane is concave down, then its curvature is negative.

(g) True False If $\mathbf{r}(t)$ is the position of a particle at time t, then it must be true that $\operatorname{proj}_{\mathbf{B}} \mathbf{r}'' = \mathbf{0}$ at every value of t (provided $\mathbf{B} \neq \mathbf{0}$).

(h) True False If $\mathbf{r}(t)$ is the position of a particle at time t, and $\mathbf{r}' \cdot \mathbf{r}'' < 0$ at time t, then $\frac{d}{dt}|\mathbf{r}'| < 0$ at time t.

8. (3 points) Compute $\operatorname{proj}_{\langle 1,2,2\rangle} \langle 3, -1, 5\rangle$.

9. (6 points (bonus)) Compute the arc length of the curve $\mathbf{r}(t) = \langle \cos t, \sin t, \ln \cos t \rangle$, $t \in [0, \pi/4]$.

Scratch Paper - you may remove this if you find it convenient

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